

3.4 Completed Notes

3.4: Algorithms for Multiplication and Division

Example: (The Standard Algorithm - Single Digit)

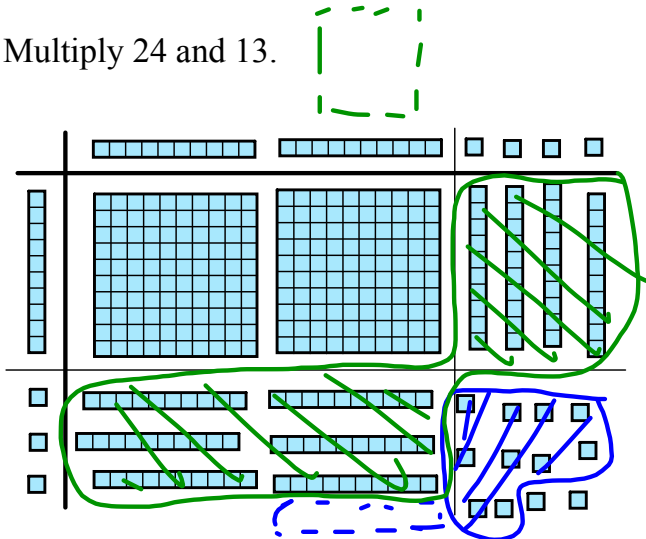
Multiply 3037 and 4.

$$\begin{array}{r}
 12 \\
 3037 \\
 3037 \\
 3037 \\
 + 3037 \\
 \hline
 12148
 \end{array}$$

$$\begin{array}{r}
 12 \\
 3037 \\
 \times 4 \\
 \hline
 12148
 \end{array}$$

Example: (The Standard Algorithm - Multiple Digit)

Multiply 24 and 13.



Standard Algorithm:

$$\begin{array}{r}
 \times 24 \\
 13 \\
 \hline
 172 \\
 + 240 \\
 \hline
 312
 \end{array}$$

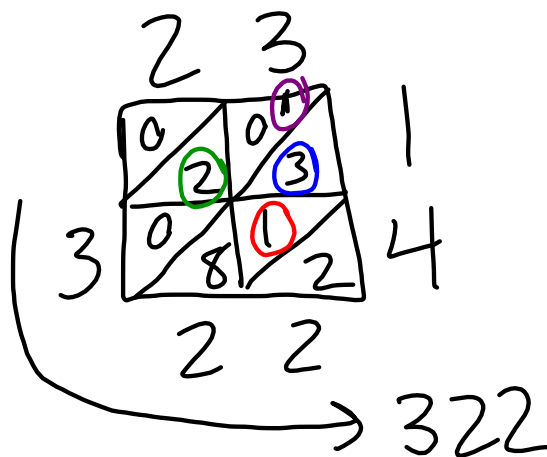
3.4 Completed Notes

Another Explanation:

$$(20+3)(10+4)$$
$$\begin{array}{r} 23 \\ \times 14 \\ \hline \end{array} = \begin{array}{r} 20 + 3 \\ \times 10 + 4 \\ \hline 80 + 12 \\ + 200 + 30 \\ \hline 280 + 42 = 322 \end{array}$$

Example: (Lattice Multiplication)

(a) Multiply 23 and 14.



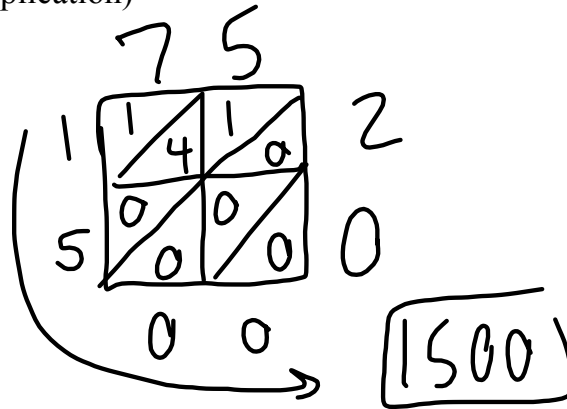
Why does this work?

- regrouped 3×4 into 1 ten + 2 units
- actually multiplied 3×10
- actually multiplied 20×10
- regrouped 12 tens into 1 hundred and 2 tens

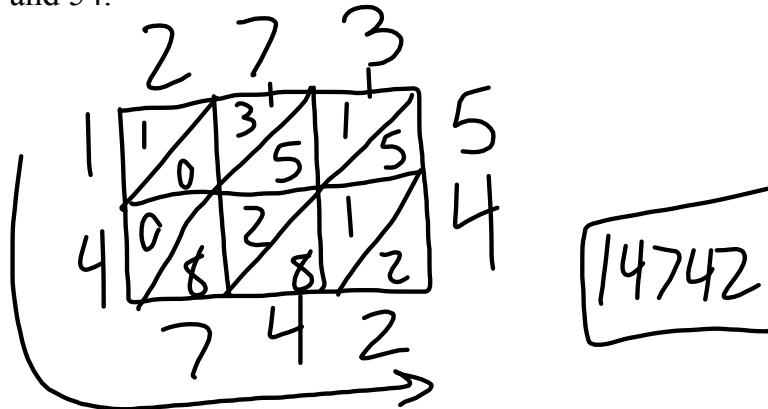
3.4 Completed Notes

Example: (Lattice Multiplication)

(b) Multiply 75 and 20.



(c) Multiply 273 and 54.

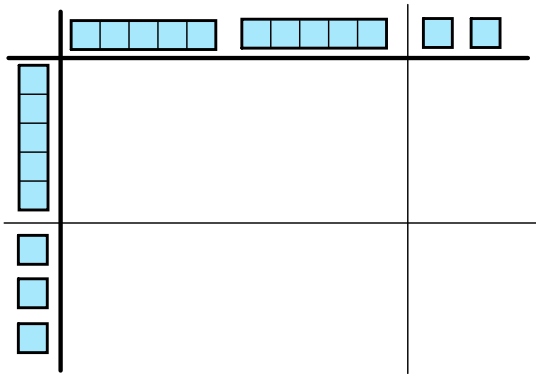


Base 5 Multiplication: We can use both of the previous algorithms with base 5 numbers using this table.

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

3.4 Completed Notes

Example: Multiply 14_{five} and 22_{five} .



Standard Algorithm:

$$\begin{array}{r}
 1 \\
 14 \\
 \times 22 \\
 \hline
 33 \\
 +330 \\
 \hline
 413_{\text{five}}
 \end{array}$$

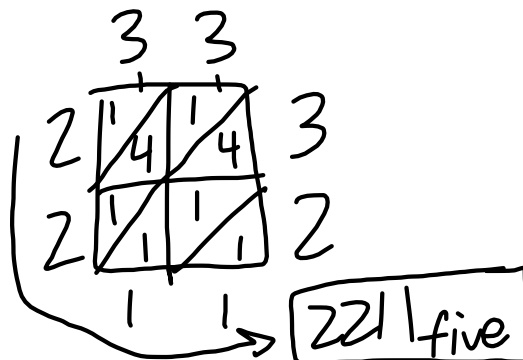
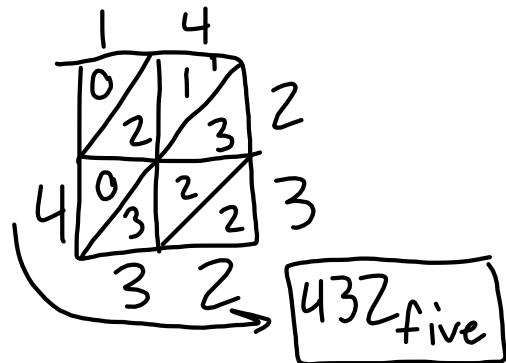
Example: All numbers are base 5.

Standard:

$$\begin{array}{r}
 1 \\
 \times 14 \\
 \hline
 102 \\
 +330 \\
 \hline
 432_{\text{five}}
 \end{array}$$

$$\begin{array}{r}
 1 \\
 \times 33 \\
 \hline
 121 \\
 +2040 \\
 \hline
 2211_{\text{five}}
 \end{array}$$

Lattice:



3.4 Completed Notes

The Long Division Algorithm:

Example: Calculate $617 \div 5$

The diagram illustrates the long division of 617 by 5. It consists of several parts:

- Grids:** A 10x10 grid representing 617 units. The first two full grids (200 units) and the first 17 units of the third grid are crossed out with red 'X's. The remaining 17 units are crossed out with green 'X's.
- Long Division:**

$$\begin{array}{r} 123R2 \\ 5 \overline{) 617} \\ \underline{-5} \\ 11 \\ \underline{-10} \\ 17 \\ \underline{-15} \\ 2 \end{array}$$
- Place Value Chart:** A vertical bar on the left with a blue 'X' and a green 'X' next to it. To the right, a circle contains two squares and two dots, with the word "remainder" written above it.
- Units:** Five circles at the bottom, each containing a red square, a green square, and two blue dots, representing the quotient 123.

Example: Carefully explain why this works:

$$\begin{array}{r} 86 \\ 23 \overline{) 1986} \\ \underline{-184} \\ 146 \\ \underline{-138} \\ 8 \end{array}$$

- ① Form 23 groups
- ② Can't put 1 cube in 23 groups, so form 10 flats, giving 19 flats.
- ③ Convert 19 flats into 190 longs, giving 198 longs
- ④ Put 8 longs in each of the groups. You use 184 longs and leave 14 longs.
- ⑤ Convert the 14 longs into 140 units, giving 146 units.
- ⑥ Put 6 units in each of the 23 groups. You use 138 units and leave the remainder of 8.

3.4 Completed Notes

Example: Calculate $342_{\text{five}} \div 2_{\text{five}}$.

$$\begin{array}{r} \boxed{143}_{\text{five}} \text{ R } \boxed{1}_{\text{five}} \\ 2 \overline{) 342} \\ \underline{-2} \\ 14 \\ \underline{-13} \\ 2 \\ \underline{-11} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 11 \end{array} \quad \begin{array}{r} 2 \\ \times 4 \\ \hline 13 \end{array}$$

Let's try $342_{\text{five}} \div 2_{\text{five}}$ with Base 5 blocks.

3.4 Completed Notes

Example: Calculate $213_{\text{five}} \div 3_{\text{five}}$.

$$\begin{array}{r}
 \boxed{34_{\text{five}} \text{ R } 1_{\text{five}}} \\
 3 \overline{) \cancel{2} \cancel{1} 3} \\
 \underline{-14} \downarrow \\
 23 \\
 \underline{-22} \\
 1
 \end{array}
 \qquad
 \begin{array}{r}
 3 \\
 \times 2 \\
 \hline
 11
 \end{array}
 \qquad
 \begin{array}{r}
 3 \\
 \times 3 \\
 \hline
 14
 \end{array}
 \qquad
 \begin{array}{r}
 3 \\
 \times 4 \\
 \hline
 22
 \end{array}$$

Example: Calculate $1322_{\text{five}} \div 32_{\text{five}}$.

$$\begin{array}{r}
 \boxed{22_{\text{five}} \text{ R } 13_{\text{five}}} \\
 32 \overline{) \cancel{1} \cancel{3} 22} \\
 \underline{-114} \downarrow \\
 132 \\
 \underline{-114} \\
 13
 \end{array}
 \qquad
 \begin{array}{r}
 32 \\
 \times 2 \\
 \hline
 114
 \end{array}
 \qquad
 \begin{array}{r}
 1 \\
 32 \\
 \times 3 \\
 \hline
 201
 \end{array}$$

3.4 Completed Notes

Example: Calculate $2002_{\text{five}} \div 21_{\text{five}}$.

$$\begin{array}{r}
 \boxed{42_{\text{five}} \text{ R } 20_{\text{five}}} \\
 21 \overline{) 2002} \\
 \underline{-134} \\
 112 \\
 \underline{-42} \\
 20
 \end{array}$$

$ \begin{array}{r} 21 \\ \times 2 \\ \hline 42 \end{array} $	$ \begin{array}{r} 21 \\ \times 3 \\ \hline 113 \end{array} $	$ \begin{array}{r} 21 \\ \times 4 \\ \hline 134 \end{array} $
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$3424_{\text{five}} \div 23_{\text{five}}$

$$\begin{array}{r}
 \boxed{122_{\text{five}} \text{ R } 13_{\text{five}}} \\
 23 \overline{) 3424} \\
 \underline{-23} \downarrow \\
 112 \\
 \underline{-101} \\
 114 \\
 \underline{-101} \\
 13
 \end{array}$$

$ \begin{array}{r} 1 \\ 23 \\ \times 2 \\ \hline 101 \end{array} $	$ \begin{array}{r} 1 \\ 23 \\ \times 3 \\ \hline 124 \end{array} $	$ \begin{array}{r} 2 \\ 23 \\ \times 4 \\ \hline 202 \end{array} $
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