3.4: Algorithms for Multiplication and Division

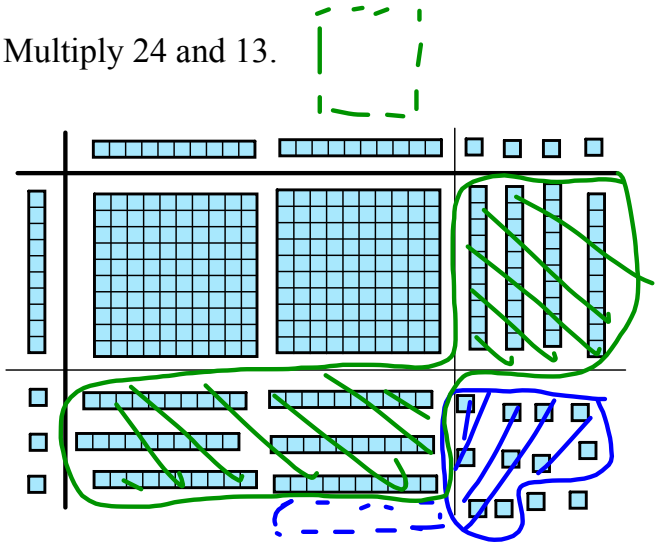
Example: (The Standard Algorithm - Single Digit)

Multiply 3037 and 4.

$$\begin{array}{r} {\scriptstyle 1\ 2\ \ \ } \\ 3\ 0\ 3\ 7 \\ 3\ 0\ 3\ 7 \\ 3\ 0\ 3\ 7 \\ +\ 3\ 0\ 3\ 7 \\ \hline 12148 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 2\ \ \ } \\ 3\ 0\ 3\ 7 \\ \times \qquad 4 \\ \hline 12148 \end{array}$$

Example: (The Standard Algorithm - Multiple Digit)

Multiply 24 and 13.



Standard Algorithm:

$$\begin{array}{r} 2\ 4 \\ \times\ 1\ 3 \\ \hline 72 \\ +240 \\ \hline 312 \end{array}$$

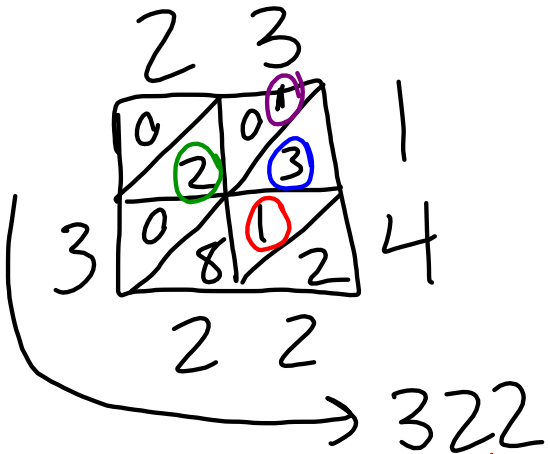
Another Explanation:

$$(20+3)(10+4)$$

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline \end{array} = \begin{array}{r} 20+3 \\ \times\ 10+4 \\ \hline 80+12 \\ +200+30 \\ \hline 280+42=322 \end{array}$$

Example: (Lattice Multiplication)

(a) Multiply 23 and 14.

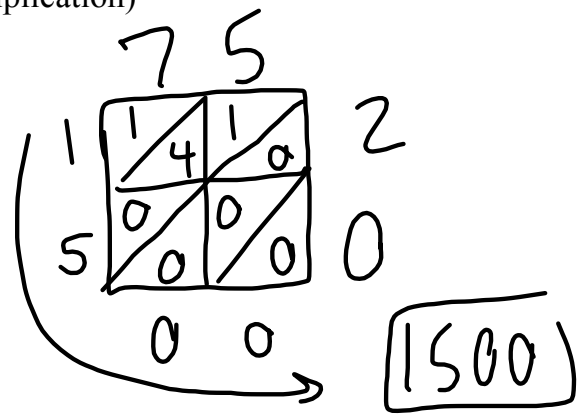


Why does this work?

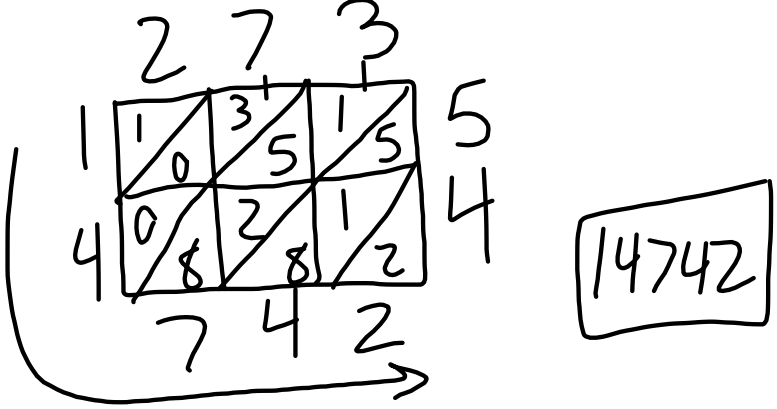
- regrouped 3×4 into 1 ten + 2 units
- actually multiplied 3×10
- actually multiplied 20×10
- regrouped 12 tens into 1 hundred and 2 tens

Example: (Lattice Multiplication)

(b) Multiply 75 and 20.



(c) Multiply 273 and 54.

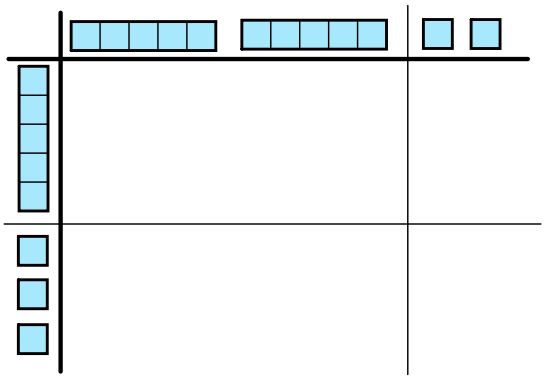


Base 5 Multiplication: We can use both of the previous algorithms with base 5 numbers using this table.

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

3.4 Completed Notes

Example: Multiply 14_{five} and 22_{five}.



Standard Algorithm:

$$\begin{array}{r} {\scriptstyle 1} \\ 14 \\ \times\ 22 \\ \hline 33 \\ +330 \\ \hline 413_{five} \end{array}$$

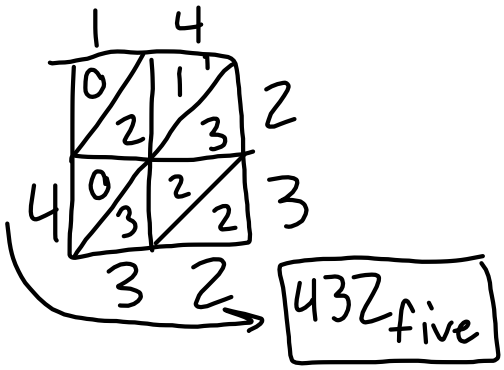
Example: All numbers are base 5.

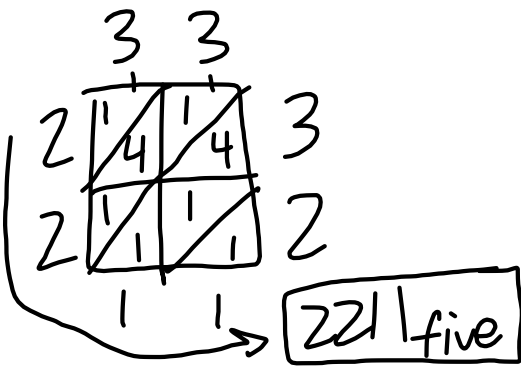
Standard:

$$\begin{array}{r} {\scriptstyle 1} \\ 14 \\ \times\ 23 \\ \hline 102 \\ +330 \\ \hline 432_{five} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1} \\ 33 \\ \times\ 32 \\ \hline 121 \\ +2040 \\ \hline 2211_{five} \end{array}$$

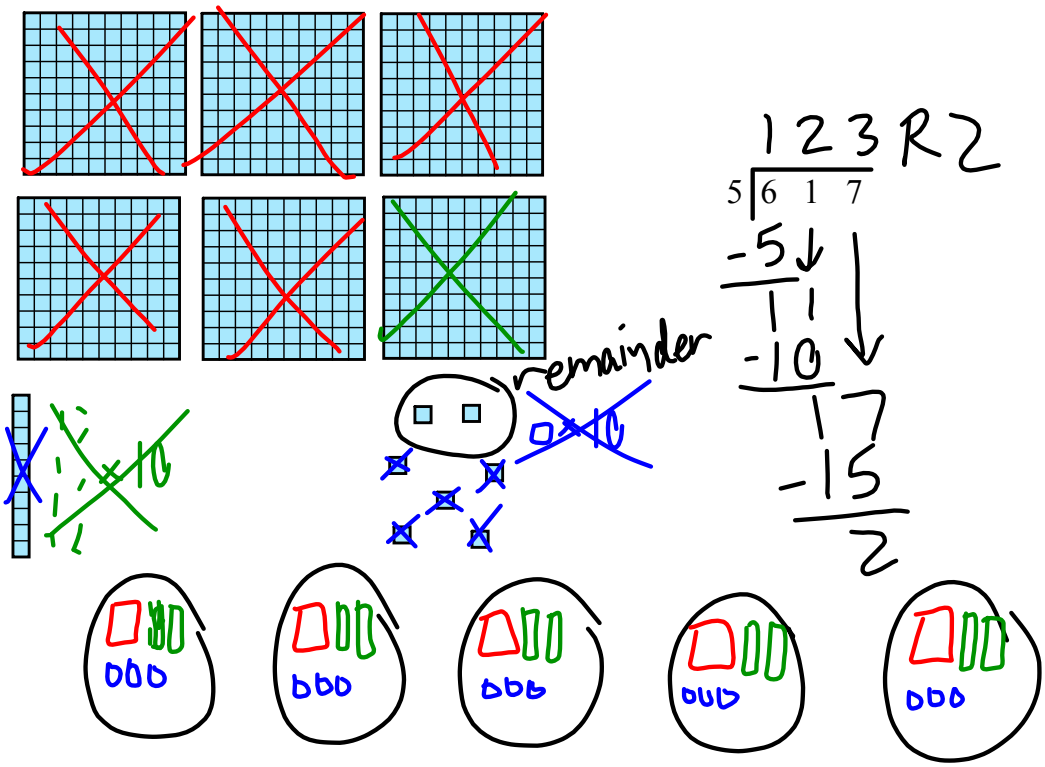
Lattice:





The Long Division Algorithm:

Example: Calculate 617 ÷ 5



$$\begin{array}{r} 123\ R2 \\ 5\overline{)617} \\ -5\downarrow \\ \hline 11 \\ -10 \\ \hline 17 \\ -15 \\ \hline 2 \end{array}$$

Example: Carefully explain why this works:

$$\begin{array}{r} 86 \\ 23\overline{)1986} \\ -184\downarrow \\ \hline 146 \\ -138 \\ \hline 8 \end{array}$$

① Form 23 groups

② Can't put 1 cube in 23 groups, so form 10 flats, giving 19 flats.

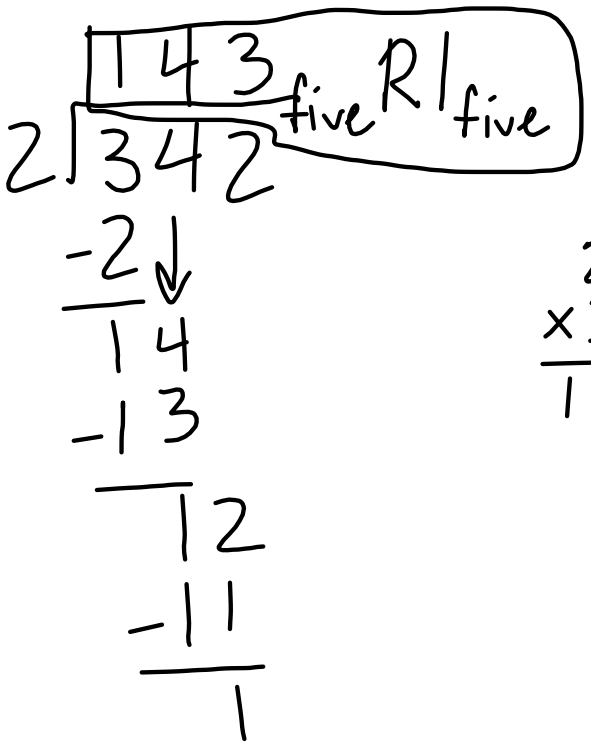
③ Convert 19 flats into 190 longs, giving 198 longs

④ Put 8 longs in each of the groups. You use 184 longs and leave 14 longs.

⑤ Convert the 14 longs into 140 units, giving 146 units.

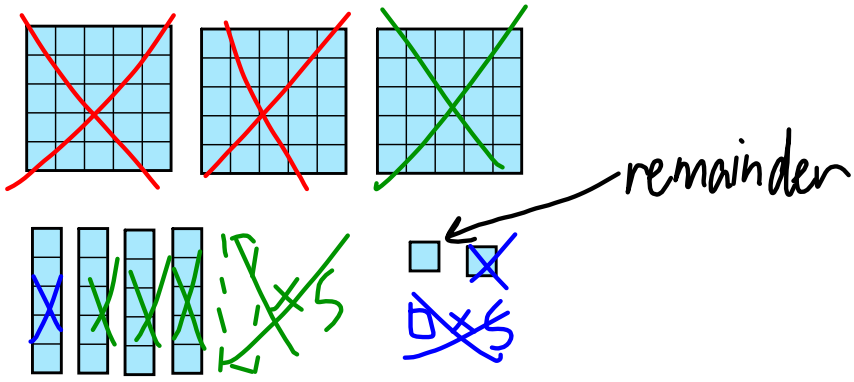
⑥ Put 6 units in each of the 23 groups. You use 138 units and leave the remainder of 8.

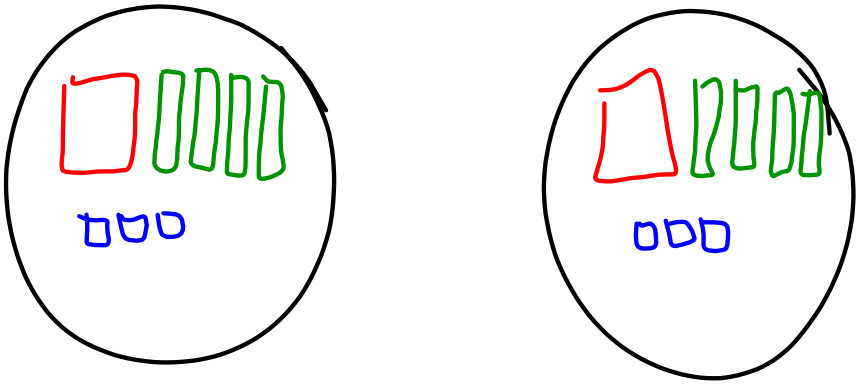
Example: Calculate $342_{five} \div 2_{five}$.



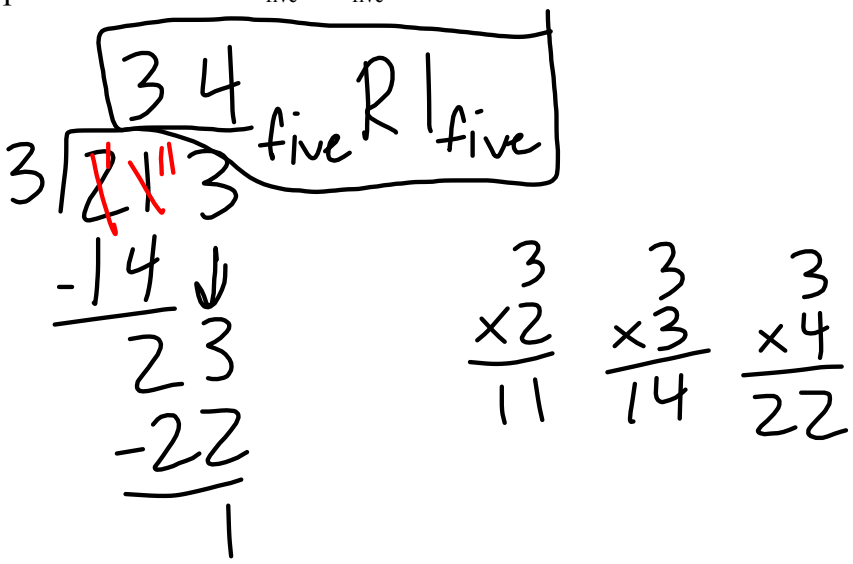
$$\begin{array}{r} 2 \\ \times 3 \\ \hline 11 \end{array} \qquad \begin{array}{r} 2 \\ \times 4 \\ \hline 13 \end{array}$$

Let's try $342_{five} \div 2_{five}$ with Base 5 blocks.

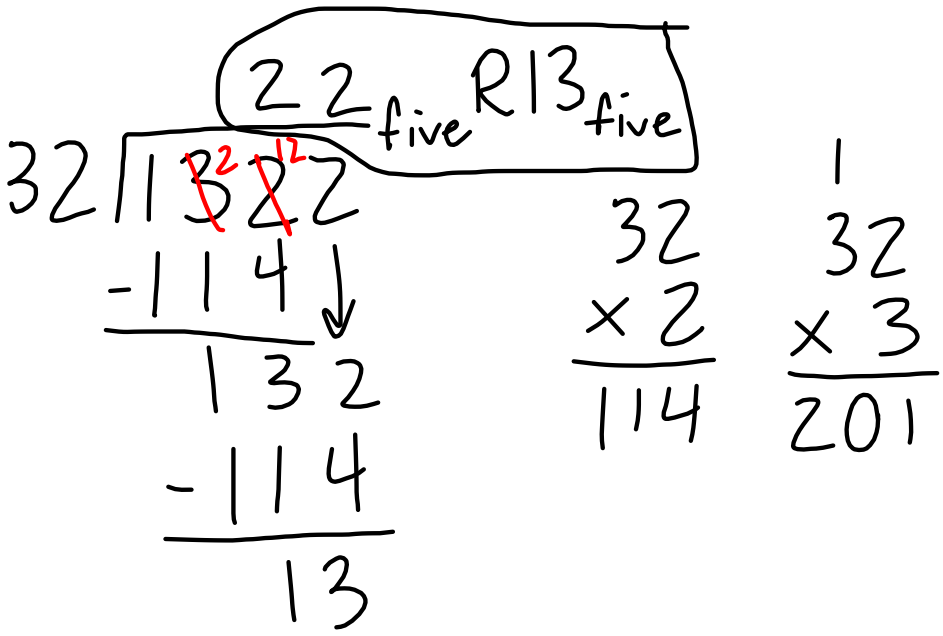




Example: Calculate $213_{five} \div 3_{five}$.



Example: Calculate $1322_{five} \div 32_{five}$.



Example: Calculate $2002_{five} \div 21_{five}$.

$42_{five} R 20_{five}$

$21 \overline{)2002}$

-134

112

-42

20

$21 \times 2 = 42$

$21 \times 3 = 113$

$21 \times 4 = 134$

$3424_{five} \div 23_{five}$

$122_{five} R 13_{five}$

$23 \overline{)3424}$

-23

112

-101

114

-101

13

$23 \times 2 = 101$

$23 \times 3 = 124$

$23 \times 4 = 202$